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ABSTRACT

High multiplicity hadronic collisions could lead to the formation of a transient classical pion field. The soft pions produced in the decay of such a classical field exhibit characteristic and measurable correlations. To allow for a discussion of such questions on a concrete example, we present an analytic solution of the classical non linear $\sigma$ model with boundary conditions appropriate to hadronic collisions at asymptotically high energies. A short discussion of various other approaches being presently developed is also given.

In this talk, I speculate on the possibility that a classical pion field be produced in the course of a high energy hadron-hadron or nucleus-nucleus collision. Such phenomenon would be signaled by characteristic fluctuations in the ratio of neutral to charge pions, perhaps not unlike those of the Centauro type events observed in cosmic rays[1]. There is presently a growing interest in such questions, as reflected by the rapidly increasing number of papers on the subject. I shall give a brief account at the end of the talk of some of the various approaches which are presently being explored. However, most of my discussion will be centered on a concrete model, namely the classical non linear sigma model, for which an analytical solution has been obtained recently for boundary conditions appropriate to high energy collisions[2].

The starting point of our discussion is the remark that high energy hadronic or nuclear collisions produce many pions, so many that the typical number of pion quanta per unit cell of phase space may substantially exceed unity. To see that, consider a Pb-Pb collision at RHIC, i.e. at center of mass energy $\sqrt{s} = 200$ GeV per nucleon. One expects in such collisions a typical multiplicity of about 700-2000 pions per unit rapidity in the central region, that is between 200 and 700 identical pions per unit rapidity[3]. These pions occupy, at time $t$, a spatial volume $V_r(t) = \pi r_0^2 A^{2/3} t \Delta y$ and, a momentum space volume $V_p = 2\pi p_T dp_T p_0 \Delta y$. Assuming a standard transverse momentum space distribution $\mathrm{d}N/\mathrm{d}p_T^2 = 2N/\langle p_T\rangle^2 \exp\left(-2p_T/\langle p_T\rangle\right)$, one gets for the average phase-space density of the pions the estimate:

$$\bar{f} = \frac{2Nh^3}{V_r(t) V_p \langle p_T\rangle^2} \mathrm{e}^{-\frac{2p_T}{\langle p_T\rangle}},$$

which, for $\langle p_T\rangle \approx p_0 \approx 2m_\pi$, reduces to

$$\bar{f} \approx 2\pi \left(\frac{\lambda_\pi^3}{r_0^2 t \Delta y}\right) \mathrm{e}^{-\frac{p_T}{m_\pi}} \frac{1}{A^{2/3}} \frac{\mathrm{d}N}{\mathrm{d}y}. \tag{1}$$

where $\lambda_\pi = \hbar/m_\pi$ is the pion Compton wavelength. For $p_T \approx 200$ MeV, one finds $\bar{f} \approx 1.5 A^{-2/3} \mathrm{d}N/\mathrm{d}y$, at time $t = 1$ fm/c, which for Pb-Pb collision, is a large number, between 10 and 30. Furthermore, since $\bar{f}$ decreases with time as $1/t$, this number remains larger than 1 for quite some time, $t \gtrsim 10$ fm/c.

Under such circumstances, one may speculate about the possibility that, in the early stages of the collision, a classical pion field develops, before subsequently decaying into individual pions. It is worth emphasizing that a large density of pions in phase space is by no means a sufficient condition (nor, perhaps, a necessary one) for creating a classical field, but it suggests that it is not unreasonable to attempt a semi-classical description of the phenomenon.

A simple model which describes pions and their interactions, and represents well much of the low energy physics of QCD, is the non linear sigma model. Its Lagrangian can be written in the form

$$\mathcal{L} = \frac{f_\pi^2}{2} \left[(\partial\sigma)^2 + (\partial\vec{\pi})^2\right] \tag{2}$$

where $f_\pi = 93$ MeV is the pion decay constant and the fields $\sigma$ and $\vec{\pi}$ satisfy the constraint $\sigma^2 + \vec{\pi}^2 = 1$. In Ref.2, an analytic solution of the classical equations of motion resulting from (2) has been obtained, for boundary conditions appropriate to hadronic collisions at extremely high energy. One assumes that, initially, the whole energy of the collision is contained in an infinitesimally thin slab of infinite transverse extent, reducing the problem effectively to a 1+1 dimensional one. In this particular situation, the fields depend only on the "proper time" $\tau = \left(t^2 - z^2\right)^{1/2}$, where $z$ is the longitudinal coordinate.

The solution depends on two arbitrary orthogonal isovectors $\vec{a}$ and $\vec{b}$, which are constants of the motion, and to which the isovector and isoaxial currents are proportional:

$$\vec{V}_\mu = \vec{\pi} \times \partial_\mu \vec{\pi} = \vec{a}\partial_\mu \phi \qquad \vec{A}_\mu = \vec{\pi}\partial_\mu \sigma - \sigma \partial_\mu \vec{\pi} = \vec{b}\partial_\mu \phi, \tag{3}$$

where the scalar function $\phi(\tau)$ satisfies the wave equation $\partial^2\phi = 0$. The $\vec{\pi}$ fields have no component on $\vec{a}$, and "rotate" in the plane spanned by $\vec{b}$ and $\vec{a} \times \vec{b} \equiv \vec{c}$, according to

$$\begin{cases} \pi_a &= 0 \\ \pi_b &= -\sin\left(2\kappa \ln\frac{\tau}{\tau_0}\right) \\ \pi_c &= \frac{a}{\kappa}\cos\left(2\kappa \ln\frac{\tau}{\tau_0}\right) \end{cases} \tag{4}$$

where $\kappa^2 = a^2 + b^2$. Note that this solution agrees with the general form written by Anselm[4]. However Anselm and Ryskin used different boundary conditions in their subsequent physical analysis[4,5], so their explicit solution differs from ours.

When $\tau \gg \tau_0$, the fields wildly oscillate. This may be understood by noticing that, at any fixed time, the region in space-time between the light cone $z = \pm t$ and the hyperbola $\tau = \tau_0$, contains an infinite amount of energy. Since the amplitude of the pion field is bounded, the only way to store a large amount of energy in the field is by producing these violent oscillations. We do not regard these oscillations as physical: clearly, in the region where they occur, high-order derivative terms ought to be taken into account in the effective Lagrangian. On the other hand, it is tempting to speculate that this region of violent oscillations represents the high energy density region where one expects chiral symmetry to be restored. In fact, the averages of the fields over small time interval vanish for small enough $\tau$, mimicking chiral symmetry restoration.

Having an explicit solution at hand, we can calculate various observables. For example, the energy momentum tensor takes the form

$$T_{\mu\nu} = S\left(2u_\mu u_\nu - g_{\mu\nu}\right)\varepsilon \tag{5}$$

where $S$ is a transverse area, $u_\mu = x_\mu/\tau$ and $\varepsilon = 2f_\pi^2\kappa^2/\tau^2$ is the energy density of the field configuration. One can estimate the energy radiated at time $t$ in the following way. We write

$$\int \mathrm{d}z T_{00}(z,t) = \int \mathrm{d}k \frac{\mathrm{d}E}{\mathrm{d}k}. \tag{6}$$

Both integrals in (6) diverge (because there is an infinite amount of energy stored in the vicinity of the light cone, or equivalently in high momentum modes); however $\mathrm{d}E/\mathrm{d}k$ is finite and independent of $t$ :

$$\frac{\mathrm{d}E}{\mathrm{d}k} = S\frac{f_\pi^2\kappa}{\tanh \pi\kappa} \tag{7}$$

Thus the spectrum of soft pions exhibits the familiar rapidity plateau $\mathrm{d}N/\mathrm{d}y = S\ f_\pi^2 \kappa / \tanh\ \pi\kappa$.

The energy density decreases rapidly with $\tau$. After a time $\tau_0 \approx \sqrt{2} f_\pi \kappa / m_\pi^2$, which one may call a "pion formation time", it takes the value $\varepsilon(\tau_0) \approx m_\pi / \lambda_\pi^3$ corresponding to one pion at rest in any cell having the size of one pion Compton wavelength. The value of $\tau_0$ is proportional to $\kappa$ and grows with multiplicity. At CERN SPS one expects $\kappa \gtrsim 5$ so that $\tau \gtrsim 7$ fm/c$^2$. This number indicates that, if produced, such a classical field would live a rather long time before decaying into individual pions. However this estimate is based on the, presumably unrealistic, assumption that in a nucleus-nucleus collision, the source is totally coherent over its entire transverse extent.

The solution (4) depends on the two isovectors $\vec{a}$ and $\vec{b}$, which are proportional to the vector and axial currents respectively. In fact, from (3) and (4) we get

$$\vec{V}_\mu = 2x_\mu \frac{\vec{a}}{\tau^2} \qquad \vec{A}_\mu = 2x_\mu \frac{\vec{b}}{\tau^2} \tag{8}$$

The vectors $\vec{a}$ and $\vec{b}$ are constants of the motion. Therefore we are led to assume that the directions of the isovector and isoaxial currents are, in a sense, frozen at the early stage of the collision, and remain fixed during the subsequent evolution of the system. This peculiar feature of the classical field configuration implies large fluctuations in the observed ratio of neutral to charged particles. Indeed, depending on the orientation $\vec{a}$ and $\vec{b}$, the decay of the classical field can lead to mostly neutral pions, or to only charged pions. Of course, all directions in isospace are, a priori, equally probable. One can use this property to show, using a simple geometrical argument, that the probability that neutral pions constitute a fraction $r$ of all soft pions produced in the collision is

$$\mathrm{d}P(r) = \frac{\mathrm{d}r}{2\sqrt{r}} \tag{9}$$

This is quite distinct from an ordinary binomial distribution peaked at the value 1/3. To appreciate the content of eq.(9), let us remark that the probability to observe $r < r_0$ is $\sqrt{r_0}$, so that in 10% of the events the ratio $r$ is less than 0.01%; similarly, in 5% of the events, $r$ is bigger than 0.9%.

It is worth emphasizing at this point that the magnitude of the effect to be possibly observed will depend on the size of the regions over which the sources are correlated. For example, in a nucleus-nucleus collision, the regions of the initial dense system that are sufficiently well separated in the transverse direction may not act coherently, but rather independently. Thus, the orientation of the vectors $\vec{a}$ and $\vec{b}$ may fluctuate from place to place in the transverse plane, thereby reducing, perhaps destroying, the expected correlations.

More detailed correlations can be studied. Indeed, classical field configurations describing systems with internal symmetries often exhibit correlations between orientations in coordinate space and internal space. A familiar example is provided by the Skyrme model of the nucleon, in which the isospin of the pion field at point $\vec{r}$ is aligned with the radial vector $\vec{r}$, in the so-called "hedgehog" configuration. One can envisage similar properties for the classical pion field produced in a nucleus-nucleus collision. In 6, we have assumed cylindrical symmetry and isotopic invariance of the colliding system, and considered classical pion field configurations of the form

$$\pi_a(x) = O^{ai} F_i(x) \tag{10}$$

where $O^{ai}$ is an orthogonal matrix relating the isospin orientation $a$ to the orientation $i$ of the vector field $F_i(x)$. With this ansatz one can easily derive specific predictions for the inclusive two-particle cross sections, from which well defined azimutal correlations between pions can be deduced. For illustration, let us quote one example which involves a particular combination of cross sections $\sigma^{ij}$ for producing a pair of pions of types $i$ and $j$, with zero rapidity[6] :

$$\frac{4\sigma^{+-} - 3\sigma^{+0}}{2\sigma^{+0} + \sigma^{00}} = \cos^2\left(\phi_1 - \phi_2\right)$$

where $\phi_1$ and $\phi_2$ are the azimuthal angles of the pions.

Most of these correlations, and especially the probability (9), are fairly general, and do not depend on the detailed mechanism by which the pion field is created. Similar correlations would be obtained with various ansatz for the quantum states associated with the classical field configurations[7,8,9]. Such quantum states are basically coherent states, or more properly squeezed states[8]. An interesting question one may ask is how long such a state, once produced, can remain coherent. This issue has been recently studied by Krzywicki who considered a coherent state coupled with a heat bath (formed by the debris of the collision), and identified the decoherence time as the time it takes for the density matrix to become diagonal in Fock space.

Various mechanisms have been proposed as possibly leading to the formation of a classical pion field, or a pion coherent state. In 2 the *radiation* aspect of the problem was emphasized, following in spirit the early work of Heisenberg[11].

In other approaches, the pion field is generated in a *relaxation* process from a state of "misaligned vacuum" or "disoriented chiral condensate"[12,13]. In Bjorken's scenario[12], one assumes that after a violent hadronic collision, there is a "cool" region surrounded by a "hot" expanding shell separating the cool region from outer space. As the cool region emerges from a chirally symmetric state, and is physically isolated from the true vacuum by the collision debris, it may evolve to a "wrong" vacuum state (the $\sigma$ field pointing into the wrong direction). When the whole system further cools down, it eventually has to relax towards the true vacuum, and this relaxation may be accompanied by the formation of large amplitude classical pion waves. Concrete simulations of a similar relaxation process have been carried out recently by Rajagopal and Wilczek[13]. Of course, the magnitude of the classical field depends very much on the size of the domains in which the chiral condensate is disoriented[14].

The two mechanisms that I have discussed, namely radiation and relaxation, are genuine off-equilibrium effects. There has been suggestions in the past that a pion coherent state, or *pion condensate*, could be produced in equilibrium, or close to equilibrium, situations, as a result of attractive interactions[15]. It not clear whether such pion condensates have anything to do with the classical fields produced by the dynamical mechanisms mentioned above. In fact, recent analysis seem to imply that it is difficult to produce them in a nucleus-nucleus collision if matter stays in local thermal equilibrium[9].

Clearly, I have only superficially touched some of the many interesting issues which can be addressed in this field. Much work remains to be done in order to specify under which conditions such new and fascinating collective phenomena could be observed.

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